

The University of Alabama  
Thirty-first Annual  
High School Mathematics Tournament  
Team Competition

October 20, 2012

1. Do **not** turn this page until the proctor indicates that it is time to begin.
2. The notation  $(f \circ g)(x)$  refers to composition of functions:  $(f \circ g)(x) = f(g(x))$ .
3. Throughout the test, the letter  $i$  represents the imaginary unit  $i = \sqrt{-1}$ ,  $\log(x)$  means  $\log_{10}(x)$ , and  $\ln(x)$  means  $\log_e(x)$ .
4. All answers must be exact, unless specifically asked to do otherwise. Leave  $\pi$ ,  $e$ , and radicals in the answer.
5. If  $M$  is a matrix,  $M^T$  is the transpose of  $M$  (columns become rows, rows become columns).
6. The test is 45 minutes in length. If you must leave the room, you **MAY NOT** re-enter the room before time is called.
7. Answers to the questions must be entered on the correct line of the answer sheet. Each question will be worth 1 point (12 points for the entire test) and no partial credit will be given. (Only the answer sheet will be turned in and graded.)
8. The overall team competition score will be calculated by adding the points for the team test (12 possible) to the points from the team participants individual percentage correct test scores (6 possible).
9. Hand-held calculators of any type are allowed. Internet access will not be allowed.

1. What is the remainder when  $x^4 + 3x^2 + x - 1$  is divided by  $x^2 + 2$ ?

2. Square ABCD is inscribed in a circle O (that is, A, B, C, and D all lie on the circle) and its area is  $a$ . Square EFGH is inscribed in a semicircle of circle O (that is, E and F lie on a diameter and G and H lie on the circle). What is the area of square EFGH?

3. Simplify:

$$\csc^{-1}(x) + \sec^{-1}(x) + \cot^{-1}(y) + \cot^{-1}\left(\frac{1}{y}\right)$$

4. When I am as old as my father is now, I will be five times as old as my son is now. By then, my son will be eight years older than I am now. The sum of my father's age and my age is 100 years. How much older am I than my son?

5. A triangle has vertices  $A(0, 0)$ ,  $B(3, 0)$ , and  $C(3, 4)$ . If the triangle is rotated counterclockwise around the origin until  $C$  lies on the positive  $y$ -axis, find the area of the intersection of the region bounded by the original triangle and the region bounded by the rotated triangle.

6. A pyramid has a square base 6 m on a side and a height of 9 m. Find the volume of the portion of the pyramid which lies above the base and below a plane parallel to the base and 3 m above the base.

7. If  $f(x) = ax^2 + bx + c$ ,  $f(-1) = 10$ ,  $f(0) = 5$ , and  $f(1) = 4$ , find  $f(2)$ .



8. Let

$$f(x) = \frac{\sqrt{x^2 - 1}}{x}.$$

Write the domain of  $f$ , the range of  $f$ , and the domain of  $f \circ f$  in interval notation.

9. Ed drives from San Mateo to Atascadero, a distance of 197.5 mi. He starts driving at a constant speed and reduces his speed by 5 mph after each half hour of driving. If the trip takes 3 hr 20 min, how far did he travel in the first 2 hours?

10. Ed drives to work at a constant speed  $S$ . One day he is halfway to work when he immediately turns around, speeds up by 8 mph, and drives home. As soon as he is home, he turns around and drives to work at 6 mph faster than he drove home. His total driving time is exactly 67% greater than usual. Find  $S$  in mph.

11. A function  $f$  is symmetric to the origin and periodic with period 8. If  $f(2) = 3$ , what is the value of  $f(4) + f(6)$ ?

12. A farmer plants  $A$  acres of wheat one year. Each year thereafter, he harvests (removes)  $\frac{1}{4}$  of the planted acreage and then plants 1500 more acres. Does the number of acres of wheat planted approach a limiting value? If so, what is that value?

# Team Competition Answer Sheet

School Name:
Question 1:
Question 2:
Question 3:
Question 4: years
Question 5:
Question 6: $m^3$
Question 7:
Question 8: Domain of $f$ : Range of $f$ : Domain of $f \circ f$ :
Question 9: miles
Question 10: miles
Question 11:
Question 12: acres