# The University of Alabama

Thirty-third Annual

High School Mathematics Tournament

Team Competition

November 8, 2014

1. Do **not** turn this page until the proctor indicates that it is time to begin.

2. The notation $(f \circ g)(x)$ refers to composition of functions: $(f \circ g)(x) = f(g(x))$.

3. Throughout the test, the letter i represents the imaginary unit $i = \sqrt{-1}$, $\log(x)$ means $\log_{10}(x)$, and $\ln(x)$ means $\log_e(x)$.

4. All answers must be exact, unless specifically asked to do otherwise. Leave π, e, and radicals in the answer.

5. The test is 45 minutes in length. If you must leave the room, you **MAY NOT** re-enter the room before time is called.

6. Answers to the questions must be entered on the correct line of the answer sheet. Each question will be worth 1 point (12 points for the entire test) and no partial credit will be given. (Only the answer sheet will be turned in and graded.)

7. The overall team competition score will be calculated by adding the points for the team test (12 possible) to the points from the top 6 individual participants percentage correct test scores (6 possible).

8. Hand-held calculators of any type are allowed. Internet access will not be allowed.

1. A jet flies from Atlanta to London at a constant speed of 500 mph in 6 hours. A very fidgety passenger spends the entire flight walking back and forth the length of the passenger cabin at a constant speed of 3 mph. If the passenger cabin is 200 ft long, how far did the fidgety passenger walk during the flight?

2. Three circles of radius 2 are drawn in such a way that each one passes through the center of the other two. What is the area of the triangle whose vertices are the centers of the three circles?
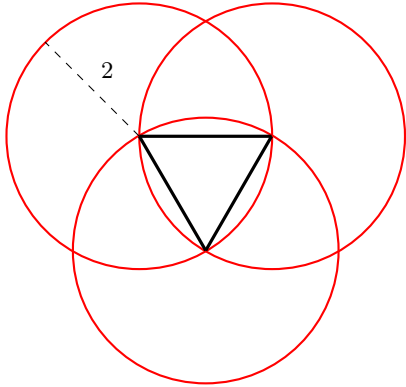
3. Solve the inequality

$$\frac{x^2 - 4x + 5}{x - 1} \le 1.$$

4. Compute $(1 + i)^{20}$ as complex number of the form $a + bi$ where a and b are real numbers.

5. What is the probability that a random positive factor of 10! will be even?

6. Find all positive real solutions to

$$\cos(\arctan(x)) = x.$$

7. The expression 3^{3^3} is ambiguous:

$$3^{(3^3)} = 3^{27} = 7625597484987$$

while

$$(3^3)^3 = 3^9 = 19683.$$

For what non-zero integer values of x is the expression x^{x^x} unambiguous, that is, find all non-zero integer solutions of

$$x^{(x^x)} = (x^x)^x.$$

8. Evaluate the continued fraction:

$$\cfrac{1}{9 + \cfrac{1}{1 + \cfrac{1}{18 + \cfrac{1}{1 + \cfrac{1}{18 + \cfrac{1}{1 + \cfrac{1}{18 + \cdots}}}}}}}$$

9. Find the first time after 3:00 pm that the hour and minute hands on a clock form a right angle. Round your answer to the nearest second.

10. A circle of radius one is inscribed in a square. In the upper right corner a second circle is inscribed. The process is repeated, inscribing the next circle in the upper right corner of the square between the previous circle. The first 3 circles are shown below. What is the radius of the 10th circle that is inscribed? It is preferred if you leave your answer in an unsimplified form as a power of an irrational number.
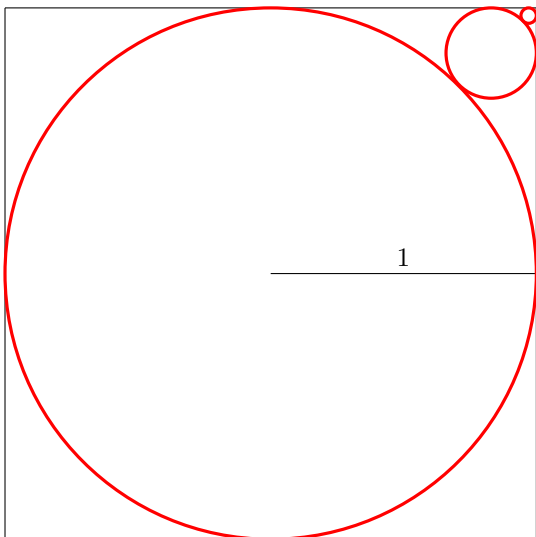
11. A woman at a point A on the shore of a circular lake with diameter 4 miles wants to arrive at the point C diametrically opposite A on the other side of the lake. She can run along the shore at a rate of 4 miles per hour or she can row a canoe at a rate of 2 miles per hour. She decides to first row to an intermediate point and then run along the shore the rest of the way. She departs point A at an angle of $30°$ from the diameter, rows in a straight line across the lake to point B, and then runs the rest of the way to C. How long will it take her to get from A to C in this manner?
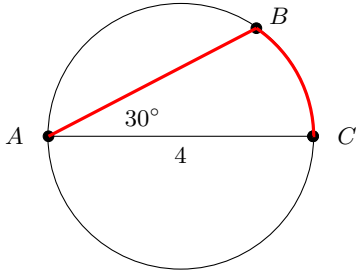
12. Three farmers, Adam, Brian, and Charlie, are selling chickens at a market. Adam has 10 chickens, Brian has 16, and Charlie has 26. In order to keep things fair, they all agree to sell their chickens at the same price. At lunchtime, they decide to (uniformly) lower their price to boost sales. At the end of the day, each farmer made \$35 and sold all of their chickens. What price were the farmers selling their chickens for in the morning? You may assume that both prices were a positive integer number of cents.

Team Competition Answer Sheet

School Name:
Question 1:   miles
Question 2:
Question 3:
Question 4:   m
Question 5:
Question 6:
Question 7:
Question 8:
Question 9:
Question 10:
Question 11:   hours
Question 12: $