

The University of Alabama

Thirty-fourth Annual

High School Mathematics Tournament

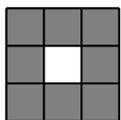
Team Competition

November 14, 2015

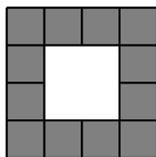
1. Do **not** turn this page until the proctor indicates that it is time to begin.
2. The notation $(f \circ g)(x)$ refers to composition of functions: $(f \circ g)(x) = f(g(x))$.
3. Throughout the test, the letter i represents the imaginary unit $i = \sqrt{-1}$, $\log(x)$ means $\log_{10}(x)$, and $\ln(x)$ means $\log_e(x)$.
4. All answers must be exact, unless specifically asked to do otherwise. Leave π , e , and radicals in the answer.
5. The test is 45 minutes in length. If you must leave the room, you **MAY NOT** re-enter the room before time is called.
6. Answers to the questions must be entered on the correct line of the answer sheet. Each question will be worth 1 point (12 points for the entire test) and no partial credit will be given. (Only the answer sheet will be turned in and graded.)
7. The overall team competition score will be calculated by adding the points for the team test (12 possible) to the points from the top 6 individual participants percentage correct test scores (6 possible).
8. Hand-held calculators of any type are allowed. Internet access will not be allowed.

1. A truck drives from Birmingham to Atlanta and back. In Atlanta, it hitches a boat trailer to the back before returning to Birmingham. On the way to Atlanta, it averaged 30 miles per gallon, but on the return trip, it averages 20 miles per gallon due to pulling the heavy load. What was the average mileage for the entire round trip?

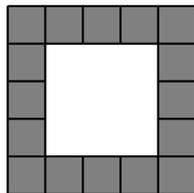
2. How many squares will be in the 8th shape?



shape 1



shape 2



shape 3

3. What is the largest closed interval $[a, b]$ on which the following equalities all hold?

$$\arcsin(\sin(x)) = x$$

$$\sin(\arcsin(x)) = x$$

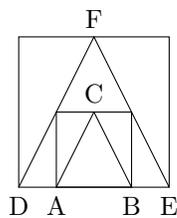
$$\arccos(\cos(x)) = x$$

$$\cos(\arccos(x)) = x$$

$$\arctan(\tan(x)) = x$$

$$\tan(\arctan(x)) = x$$

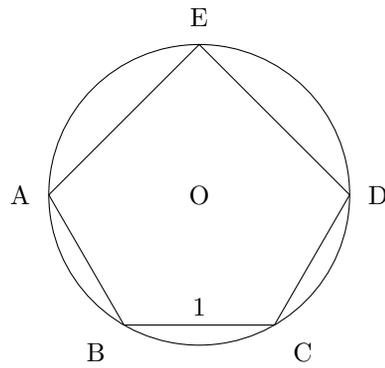
4. In the picture below, triangle ABC is inscribed in a square which is inscribed in another triangle DEF inscribed in another square. Find the ratio of the area of triangle ABC to the area of triangle DEF.



5. A triangle has sides of length 13, 14, and 15. Find the area of the triangle.

6. Susan, Jessie, Charles, and Mike are sharing four different colored hats: one red, one blue, one yellow, and one orange. Susan doesn't wear red, Jessie doesn't wear blue, Charles doesn't wear yellow, and Mike doesn't wear orange. How many different ways can they wear the four hats?

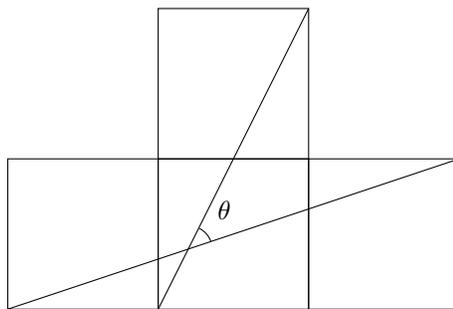
7. In the figure below, AOD forms a diameter of the circle of radius 1, centered at O , and $AE=DE$. If BC is parallel to AD and has length 1, find the area of the pentagon $ABCDE$.



8. A rectangle has length 4 and width 6. A new shape is formed by taking the set of all points that lie within one unit of a point on the boundary of the rectangle. Compute the area of this new shape.

9. When written in base 2, 2015 is a palindrome (i.e. its digits read the same forwards and backwards). What is the next year (in base 10) that is also palindromic when written in base 2?

10. The figure below consists of 4 congruent squares. Find the angle θ .



11. A line intersects the quartic $f(x) = x^4 + 3x^2 + 7x + 37$ at four distinct points. If the x -values of 3 of these points are 1, 2, and 3, find the x -value of the fourth point of intersection.

12. Let $f(x)$ be any polynomial such that $(x^2 - 2)f(x) - 1$ is divisible by x^{25} . Compute the coefficient of x^{24} of $f(x)$.

Team Competition Answer Sheet

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|--------------|
| School Name: |
| Question 1: |
| Question 2: |
| Question 3: |
| Question 4: |
| Question 5: |
| Question 6: |
| Question 7: |
| Question 8: |
| Question 9: |
| Question 10: |
| Question 11: |
| Question 12: |