# The University of Alabama

# Thirty-fifth Annual

# High School Mathematics Tournament

# Team Competition

## November 5, 2016

1. Do **not** turn this page until the proctor indicates that it is time to begin.

2. The notation $(f \circ g)(x)$ refers to composition of functions: $(f \circ g)(x) = f(g(x))$.

3. Throughout the test, the letter $i$ represents the imaginary unit $i = \sqrt{-1}$, $\log(x)$ means $\log_{10}(x)$, and $\ln(x)$ means $\log_e(x)$.

4. All answers must be exact, unless specifically asked to do otherwise. Leave $\pi$, e, and radicals in the answer.

5. The test is 45 minutes in length. If you must leave the room, you **MAY NOT** re-enter the room before time is called.

6. Answers to the questions must be entered on the correct line of the answer sheet. Each question will be worth 1 point (12 points for the entire test) and no partial credit will be given. (Only the answer sheet will be turned in and graded.)

7. The overall team competition score will be calculated by adding the points for the team test (12 possible) to the points from the team participants individual percentage correct test scores (6 possible).

8. Hand-held calculators of any type are allowed. Internet access will not be allowed.

1. If it takes Ray 72 times to fill his fish tank with a cubical container, then how many times will it take to fill up the same fish tank with a cubical container whose side length is double the original containers side length?

2. Suppose $x > 0$ and $x \neq 1$. Write the expression

$$8^{\frac{1}{\log_x(2)}}$$

as an algebraic function.

3. Write
$$\cos\left(3\arctan(x)\right)$$
as an algebraic function.

4. If the positive integers are listed in order in base 3, then what is the $28^{\text{th}}$ number in the list that only contains 0's and 1's. Write your answer in base 10.

5. Let $f(x) = 3x^7 - 7x^6 + x^3 + 4x^2 - 9x + 21$. If $A$ is the sum of the roots of $f(x)$ and $B$ be the product of the roots of $f(x)$, what is $A \times B$?

6. If $4^x + 4^{-x} = 5$ what is $8^x + 8^{-x}$?

7. Find all prime numbers $p$ so that $2p + 1$ is a perfect cube.

8. What are the last two digits (the tens and units digits) of the number $2016^{2017}$.

9. A grocery store clerk wants to make a large triangular pyramid of oranges. The bottom level is an equilateral triangle made up 1275 oranges. Each orange above the first level rests in a pocket formed by three oranges below. The stack is completed at the final level with a single orange. How many oranges are in the stack?

10. Two circles of radius $r_1$ and of $r_2$ are a distance of $d$ apart from each other. A point $P$ is to be placed on the line connecting centers of the two circles so that the tangent lines to the circles go through the point forming angles $\phi$ and $\psi$ as shown below. How far from the center of left circle should one place the point $P$ so that $\phi = \psi$? Write your answer in terms of $r_1, r_2$, and $d$.
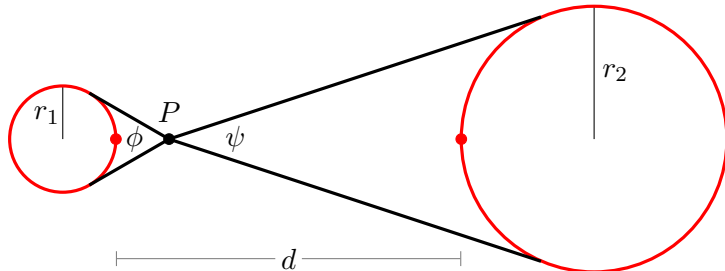
11. A $4 \times 4$ grid of points is uniformly distributed and a set of three points is to be randomly chosen from the grid. Each three point set has the same probability of being chosen. What is the probability that the three chosen points lie on the same straight line? Write your answer as a fraction in simplest form.

· · · ·

· · · ·

· · · ·

· · · ·
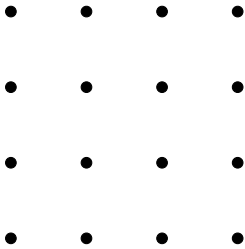
12. Suppose a circle is inscribed in an equilateral triangle with side length two meters. Another circle is inscribed in the upper corner as shown below. Find the area $A$ between the smaller circle and the corner of the triangle.

$A$

2
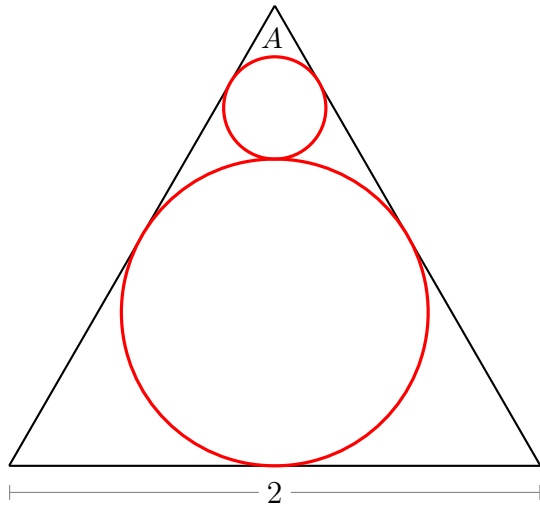
# Team Competition Answer Sheet

| |
|---|
| **School Name:** |
| **Question 1:** |
| **Question 2:** |
| **Question 3:** |
| **Question 4:** |
| **Question 5:** |
| **Question 6:** |
| **Question 7:** |
| **Question 8:** |
| **Question 9:** <div align="center">oranges</div> |
| **Question 10:** |
| **Question 11:** |
| **Question 12:** <div align="center">$m^2$</div> |