LOGARITHMIC UPPER BOUNDS FOR WEAK SOLUTIONS TO A CLASS OF PARABOLIC EQUATIONS

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Abstract. It is well known that a weak solution φ to the initial boundary value problem for the uniformly parabolic equation
\[ \partial_t \phi - \text{div}(A \nabla \phi) + \omega \phi = f \quad \text{in } \Omega_T \equiv \Omega \times (0, T) \]
 satisfies the uniform estimate
\[ \|\phi\|_{\infty, \Omega_T} \leq \|\phi\|_{\infty, \partial_p \Omega_T} + c\|f\|_{q, \Omega_T}, \quad c = c(N, \lambda, q, \Omega_T), \]
 provided that \( q > 1 + \frac{N}{2} \), where \( \Omega \) is a bounded domain in \( \mathbb{R}^N \) with Lipschitz boundary, \( T > 0 \), \( \partial_p \Omega_T \) is the parabolic boundary of \( \Omega_T \), \( \omega \in L^1(\Omega_T) \) with \( \omega \geq 0 \), and \( \lambda \) is the smallest eigenvalue of the coefficient matrix \( A \). This estimate is sharp in the sense that it generally fails if \( q = 1 + \frac{N}{2} \).

In this talk, I will begin with the history of this problem. In particular, I will describe the elegant techniques of De Giorgi and Moser. I will end with my contributions to the subject, which say that the linear growth of the upper bound in \( \|f\|_{q, \Omega_T} \) can be improved. To be precise, we establish
\[ \|\phi\|_{\infty, \Omega_T} \leq \|\phi_0\|_{\infty, \partial_p \Omega_T} + c\|f\|_{1 + \frac{N}{2}, \Omega_T} \left( \ln \left( \|f\|_{q, \Omega_T} + 1 \right) + 1 \right). \]

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