LOGARITHMIC UPPER BOUNDS FOR WEAK SOLUTIONS TO A CLASS OF PARABOLIC EQUATIONS

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Abstract. It is well known that a weak solution φ to the initial boundary value problem for the uniformly parabolic equation $\partial_t\varphi - \mathrm{div}(A\nabla\varphi) + \omega\varphi = f$ in $\Omega_T \equiv \Omega \times (0,T)$ satisfies the uniform estimate

$$\|\varphi\|_{\infty,\Omega_T} \leq \|\varphi\|_{\infty,\partial_p\Omega_T} + c\|f\|_{q,\Omega_T}, \quad c = c(N,\lambda,q,\Omega_T),$$

provided that $q > 1 + \frac{N}{2}$, where Ω is a bounded domain in $\mathbb{R}^N$ with Lipschitz boundary, $T > 0$, $\partial_p\Omega_T$ is the parabolic boundary of Ω_T, $\omega \in L^1(\Omega_T)$ with $\omega \geq 0$, and λ is the smallest eigenvalue of the coefficient matrix A. This estimate is sharp in the sense that it generally fails if $q = 1 + \frac{N}{2}$. In this talk, I will begin with the history of this problem. In particular, I will describe the elegant techniques of De Giorgi and Moser. I will end with my contributions to the subject, which say that the linear growth of the upper bound in $\|f\|_{q,\Omega_T}$ can be improved. To be precise, we establish

$$\|\varphi\|_{\infty,\Omega_T} \leq \|\varphi_0\|_{\infty,\partial_p\Omega_T} + c\|f\|_{1+\frac{N}{2},\Omega_T}\left(\ln(\|f\|_{q,\Omega_T} + 1) + 1\right).$$

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