The University of Alabama

Thirty-sixth Annual

High School Mathematics Tournament

Team Competition

November 11, 2017

1. Do **not** turn this page until the proctor indicates that it is time to begin.

2. The notation $(f \circ g)(x)$ refers to composition of functions: $(f \circ g)(x) = f(g(x))$.

3. Throughout the test, the letter i represents the imaginary unit $i = \sqrt{-1}$, $\log(x)$ means $\log_{10}(x)$, and $\ln(x)$ means $\log_e(x)$.

4. All answers must be exact, unless specifically asked to do otherwise. Leave π, e, and radicals in the answer.

5. The test is 45 minutes in length. If you must leave the room, you **MAY NOT** re-enter the room before time is called.

6. Answers to the questions must be entered on the correct line of the answer sheet. Each question will be worth 1 point (12 points for the entire test) and no partial credit will be given. (Only the answer sheet will be turned in and graded.)

7. The overall team competition score will be calculated by adding the points for the team test (12 possible) to the points from the team participants individual percentage correct test scores (6 possible).

8. Hand-held calculators of any type are allowed. Internet access will not be allowed.

1. Nick drove to his grandma's house with a speed of 30 miles per hour. He returned using the same route but with a speed of 60 miles per hour. What was his average speed?

2. Simplify the expression
$$\frac{37! - 36!}{35!}.$$

3. If $\log_a 2 = 3$, find $\log_8 a$.

4. A rectangle with side lengths of 18 in and 88 in is inscribed in a circle. What is the area of the circle? Write your answer in terms of π.

5. How many different 6 letter arrangements can be formed by using the letters *PEPPER*?
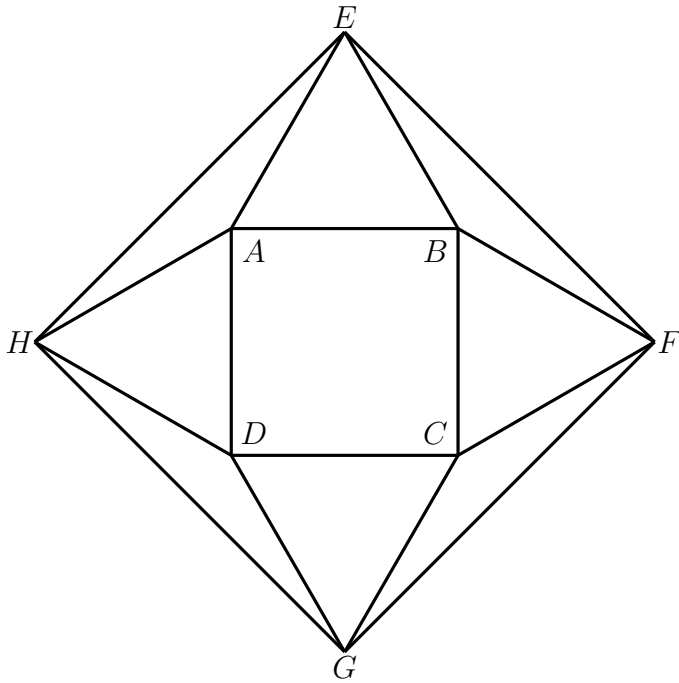
6. What is the sum of the squares of all of the roots of the polynomial

$$p(x) = x^4 - 9x^3 + 17x^2 - 8x - 11?$$

7. Write $\cos(\arctan x + \arccos x)$ as an algebraic expression.

8. Suppose $x > 1$ and satisfies $x^2 + x^{-2} = 7$. Find $x^6 - x^{-6}$.

9. Suppose that the points E, F, G, H lie in the plane of the square $ABCD$ such that AEB, BFC, CGD, and DHA are equilateral triangles. If the area of $EFGH$ is 25, then find the area of $ABCD$.

10. Evaluate the function

$$f(x) = x^3 + (x-1)^3 + (x-2)^3 + \cdots + (x-50)^3$$

when $x = 100$.

11. Take a circle of radius 1. Inscribe an equilateral triangle in it, inscribe a second circle in the triangle, inscribe a square in the second circle, inscribe a third circle in the square, inscribe a regular pentagon in the third circle, and, finally, inscribe a final circle in the pentagon. What is the radius of the final circle? You may leave your answer in terms of trigonometric functions.

12. A farmer is selling all of her sheep, goats, and cows (she has some of each). One man offers to pay her $100 for each sheep, $200 for each goat, and $400 for each cow for a total of $4700. Another offers to pay her $135 for each sheep, $265 for each goat, and $309 for each cow for a total of $5155. How many sheep, goats, and cows does she have?

Team Competition Answer Sheet

School Name:
Question 1: mph
Question 2:
Question 3:
Question 4: inches
Question 5:
Question 6:
Question 7:
Question 8:
Question 9:
Question 10:
Question 11:
Question 12: sheep   goats   cows