The University of Alabama

High School Mathematics Tournament

Team Competition

February 16, 2019

1. Do **not** turn this page until the proctor indicates that it is time to begin.
2. The notation $(f \circ g)(x)$ refers to composition of functions: $(f \circ g)(x) = f(g(x))$.
3. Throughout the test, the letter i represents the imaginary unit $i = \sqrt{-1}$, $\log(x)$ means $\log_{10}(x)$, and $\ln(x)$ means $\log_e(x)$.
4. All answers must be exact, unless specifically asked to do otherwise. Leave π, e, and radicals in the answer.
5. The test is 45 minutes in length. If you must leave the room, you **MAY NOT** re-enter the room before time is called.
6. Answers to the questions must be entered on the correct line of the answer sheet. Each question will be worth 1 point (12 points for the entire test) and no partial credit will be given. (Only the answer sheet will be turned in and graded.)
7. The overall team competition score will be calculated by adding the points for the team test (12 possible) to the points from the team participants individual percentage correct test scores (6 possible).
8. Hand-held calculators of any type are allowed. Internet access will not be allowed.

1. Two people can paint a 10 ft$\times$10 ft wall in 30 minutes. How long will it take 3 people to paint a wall that is 900 ft^2?

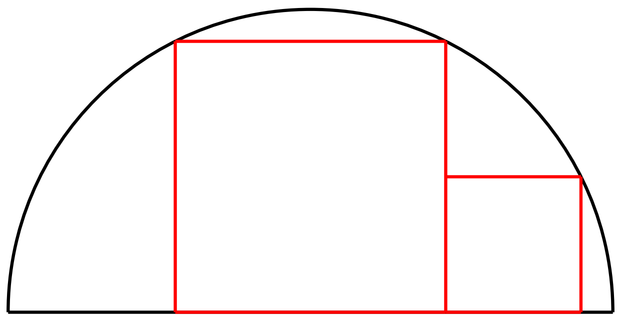
2. The mean of 10 different numbers is 20. The mean of another 18 different numbers is 6. What is the mean of all 28 numbers?

3. Suppose that $a, b, c,$ and d are numbers that satisfy $a > b > c > d > 0$ and

$$\log_4(a + c) = 3, \quad \log_3(b - c) = 2, \quad \log_2(c + d) = 1.$$

Find $a + b + c + d$.

4. Two squares are inscribed in a half-circle as shown below. If the area of the smaller square is 45, then what is the radius of the half-circle?



5. Find the exact value of $\sin(7\pi/12)$.

6. How many different 8 letter arrangements can be made using the letters ROLLTIDE such that neither of the strings ROLL or TIDE appear in the 8 letter arrangement.

7. Suppose that for all positive integers n, we have a function $f(x)$ satisfies

$$f(4+n^2) = an+2 \quad \text{and} \quad f(9-n^2) = 3n-b$$

for some numbers a and b. Find the value $f(13)$. Your answer should be a number and not in terms of a or b.

8. Find all solutions to the equation

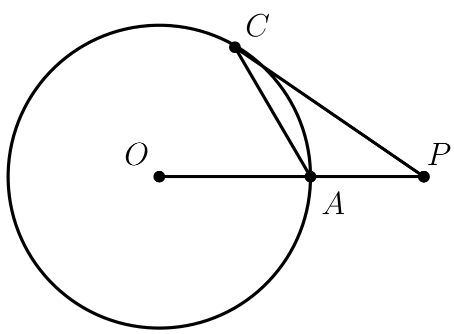
$$x^6 = 1.$$

Write complex numbers in the form $a + bi$ where a and b are real.

9. How many different ordered 4-tuples (a, b, c, d) of positive integers satisfy

$$a + b + c + d = 23?$$

10. Suppose the circle below has center O and P is a point outside the circle. Suppose further that A and C are points on the circle such that A is the intersection of the circle and the line $\overline{OP}$ and $\overline{PC}$ is tangent to the circle. If $\overline{AC} \simeq \overline{AP}$, then find the measure of the angle $\angle CPO$ in degrees.



11. Find the sum of all positive integers a such that there exists a positive integer b satisfying

$$4ab - 6a - 6b = 36.$$

12. The sum of two positive integers is 5432 and their least common multiple is 223020. Find the numbers.

Team Competition Answer Sheet

School Name:
Question 1: hours
Question 2:
Question 3:
Question 4:
Question 5:
Question 6:
Question 7:
Question 8:
Question 9:
Question 10: degrees
Question 11:
Question 12: