Thirty-Second Annual University of Alabama System Applied Mathematics Meeting

Saturday, November 16, 2019
The University of Alabama

All sessions will be held in the Gordon Palmer (GP) Hall on the campus of the University of Alabama, Tuscaloosa.

9:30am	**Refreshments**	GP 205
10:00am	**Welcome Remark**	GP 208
10:10am	**Faculty Presentation:** Hai Dang Nguyen (UA), Some New Techniques for Recurrence and Stability of Diffusion Processes in a Random Environment.	GP 208
10:50am	**Graduate Student Presentation:** Sandeep Vejandla (UAB), On iterations of certain cubic complex polynomials.	GP 208
11:10am	**Graduate Student Presentation:** Craig Bosma (UAH), On the Well-posedness and Asymptotic Behavior of the Debye-Hückel Model of Electroosmotic Flow.	GP 208
11:30am	**Lunch on your own**	Area Restaurants
1:20pm	**Faculty Presentation:** Claudio Morales (UAH), Finding fixed points for nonlinear operators in Banach spaces.	GP 208
2:00pm	**Graduate Student Presentation:** Hongsong Feng (UA), A robust and arbitrarily high order FFT Poisson solver.	GP 208
2:20pm	**Faculty Presentation:** Michael Hofbauer-Tsiflakos (UAB), The question of ergodicity for the system of falling balls.	GP 208
3:00pm	**Refreshments**	GP 205
3:20pm	**Faculty discussion**	GP 208
3:20pm	**Student discussion**	GP 206

Abstracts

On the Well-posedness and Asymptotic Behavior of the Debye-Hückel Model of Electroosmotic Flow

Craig Bosma (UAH)

Electroosmotic (EO) flow has attracted much attention in recent years due to applications in microfluidics, including chemical separation and lab-on-a-chip type systems. Because of the inherent difficulties in measuring and monitoring microfluidic channels, analysis of EO flow is often performed by numerical simulation. The Debye-Hückel model -- a simplification of the Nernst-Planck model which couples conservation of the ionic concentrations to the Navier-Stokes equations -- is the preferred model for studying such flows when the zeta potential is small.

In this work, we prove the well-posedness of the evolutionary Debye-Hückel model by Galerkin approximation, a priori estimates, and the Aubin-Simon compactness theorem for Bochner spaces. In particular, we prove the existence of solutions in two and three spatial dimensions, as well as uniqueness of the solution in 2D. We also propose and study a second-order accurate decoupled time stepping scheme based on mixed finite element spatial discretization. The time stepping scheme employs an extrapolation in time of the non-linear terms such that their skew-symmetry properties are preserved. We prove that the proposed decoupled time stepping scheme is unconditionally stable. In conclusion, we will present numerical results supporting the theoretical results.

A robust and arbitrarily high order FFT Poisson solver

Hongsong Feng (UA)

In this work, arbitrarily high order central finite differences have been applied for the first time in the literature to develop a fast solver for Poisson boundary value problems over cubic domains. The main idea is to construct a zero-padding zone outside the domain so that the fast Fourier transform (FFT) can be utilized over the entire region. By employing auxiliary variables in a Schur complement procedure, the discrete Laplacian of the central difference can be efficiently inverted by FFT, so that the overall computational efficiency of the proposed augmented matched interface and boundary (AMIB) method is about O(n^3 log n) for a n^3 grid in 3D. As a systematic approach, the AMIB method can be made to arbitrarily high order in principle, and can be easily applied in multi-dimensions. As a robust approach, the AMIB method can handle Dirichlet, Neumann, Robin boundary conditions, and their mix combinations. The accuracy, efficiency, and robustness of the proposed FFT-AMIB method are numerically validated by considering various elliptic problems in 2D and 3D.

The question of ergodicity for the system of falling balls
Michael Hofbauer-Tsiflakos (UAB)

In the late 1980's, M. Wojtkowski introduced the system of falling balls: It consists of N, N \geq 2, point masses moving up and down a vertical line, colliding with each other elastically and the lowest point mass collides with a rigid floor placed at height zero. The main question we want to answer is whether this system is ergodic or not. We are going to give a gentle review on what has been done so far and what still lies dormant in the dark.

This is a joint project with N. Simányi.

Finding fixed points for nonlinear operators in Banach spaces
Claudio Morales (UAH)

We begin discussing some old results that brings up the beauty of mathematics, as well as, the simplicity of their proofs. I must say quite shorter than earlier work done in the area. By the late sixties Browder and Kato, independently, introduced this new family of operators, called *accretive operators*. To be accurate, Kato named them as monotonic operators. Naturally, the appearance of this new family of operators evolves from the Theory of Differential Equations, but rapidly finds an interconnection with the Theory of Nonlinear Semi-groups, attaining its own self development and growth to explore whether or not certain functional equations had solutions. In this opportunity, we will entertain ourselves with an intimately related family of operators expressed as the identity operator minus an accretive one, say $I - A$, with A is accretive. In this case, the goal will be going after the existence of *fixed points*.

Some New Techniques for Recurrence and Stability of Diffusion Processes in a Random Environment
Hai Dang Nguyen (UA)

This work focuses on recurrence, ergodicity, and stability of (functional) switching diffusion consisting of continuous and discrete components, in which the discrete component takes values in a countably infinite set. Delays are also allowed. We introduce new techniques by using the log-Laplace transformation, Dupire's functional Ito formula and coupling methods. Then sufficient conditions for recurrence, ergodicity and stability of the solution process are given. Another distinctive feature is that the path-wise rate of convergence is estimated when the solution is asymptotically stable. Stabilization problems using stochastic time-delayed feedback control can also be examined.

On iterations of certain cubic complex polynomials

Sandeep Vejandla (UAB)

The goal is to parametrize the space of all cubic symmetric polynomials of the form $f(z) = z^3 +\lambda z$. We do this by studying the space of cubic symmetric laminations. In the talk, I will outline the key ideas of cubic symmetric laminations. This talk is based on joint work with Alexander Blokh, Lex Oversteegen, Nikita Selinger and Vladlen Timorin.