THE DIRICHLET PROBLEM WITH L^p DATA: WHEN CAN IT BE SOLVED?

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ABSTRACT. For a domain $\Omega \subset \mathbb{R}^d$, a classical criterion of Wiener characterizes the domains for which one can solve the Dirichlet problem (originally, for Laplace's equation) with continuous boundary data. What happens if we allow singular data, say in L^p (with respect to surface measure on the boundary) for some finite exponent p?

It turns out that solvability in the latter setting is equivalent to a quantitative, scale invariant version of absolute continuity of harmonic measure with respect to surface measure on $\partial\Omega$. In turn, in seeking to determine what sort of boundaries are permitted in the presence of such absolute continuity, one encounters a version of a classical 1-phase free boundary problem. In this talk, we shall discuss the question of characterizing L^p solvability, and we shall give an answer that is rather definitive (i.e., we find a characterization in the presence of some natural "best possible" background hypotheses), in the case of Laplace's equation. Time permitting, we will also discuss recent progress in the caloric case.

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